EE 440 Photonic Systems & Technologies 2025 – Série 1 SOLUTIONS

Exercise 1

(a) Calculate the carrier frequency corresponding to the following optical wavelengths:

The relationship between the frequency and wavelength of an electromagnetic wave is given by: $\nu = c/\lambda$. With $c = 2.997924 \cdot 10^8$ m/s. We get:

$\lambda = 633$ nm: $\nu = \frac{2.997924 \cdot 10^8}{633 \cdot 10^{-9}} = 473.6 \cdot 10^{12} = \mathbf{473.6\ THz}$

$\lambda =$800 nm: $\nu = \frac{2.997924 \cdot 10^8}{800 \cdot 10^{-9}} = 374.7 \cdot 10^{12} = \mathbf{374.7\ THz}$

$\lambda =$1300 nm: $\nu = \frac{2.997924 \cdot 10^8}{1300 \cdot 10^{-9}} = 230.6 \cdot 10^{12} = \mathbf{230.6\ THz}$

$\lambda = 1550$ nm: $\nu = \frac{2.997924 \cdot 10^8}{1550 \cdot 10^{-9}} = 193.4 \cdot 10^{12} = \mathbf{193.4\ THz}$

(b) What is the photon energy in each case? Calculate the energy in electron volt (eV) given that $1\ \mathrm{J} = 6.24 \cdot 10^{18}$ eV.

The photon energy is given by $E = h\nu$, with $h = 6.626 \cdot 10^{-34}$ $\mathrm{Js^{-1}}$. We get:

$\lambda = 633$ nm: $E = (6.626 \cdot 10^{-34})(473.6 \cdot 10^{12}) = 3.138 \cdot 10^{-19}$ J
$E = (3.138 \cdot 10^{-19})(6.24 \cdot 10^{18}) = \mathbf{1.958\ eV}$

$\lambda = 800$ nm: $E = (6.626 \cdot 10^{-34})(374.7 \cdot 10^{12}) = 2.483 \cdot 10^{-19}$ J
$E = (2.483 \cdot 10^{-19})(6.24 \cdot 10^{18}) = \mathbf{1.549\ eV}$

$\lambda = 1300$ nm: $E = (6.626 \cdot 10^{-34})(230.6 \cdot 10^{12}) = 1.528 \cdot 10^{-19}$ J
$E = (1.528 \cdot 10^{-19})(6.24 \cdot 10^{18}) = \mathbf{0.954\ eV}$

$\lambda = 1550$ nm: $E = (6.626 \cdot 10^{-34})(193.4 \cdot 10^{12}) = 1.282 \cdot 10^{-19}$ J
$E = (1.282 \cdot 10^{-19})(6.24 \cdot 10^{18}) = \mathbf{0.800\ eV}$

(c) Given that a communication system can be operated at a bit rate up to 1% of the carrier frequency find the number of audio channels at 64 kb/s that could be transmitter over a microwave carrier at $\nu_{\mu wave} = 5$ GHz or over an optical link at a wavelength of 1550 nm.

The maximum transmission rate at a carrier frequency ν_c is therefore $B_{max} = \frac{\nu_c}{100}$ bits/s.

- For the microwave carrier $\nu_c = \nu_{\mu wave} = 5$ GHz, we get:

  $$B_{max} = \frac{5 \cdot 10^9}{100} = 50 \cdot 10^6 = 50\ \mathrm{Mbit/s}$$

  The number of audio channels is therefore:

  $N_{max} = \frac{50 \cdot 10^6}{64 \cdot 10^3} \approx$ **781 channels**

- For an optical link at 1550 nm, then $\nu_c = 193.4$ THz, we get:

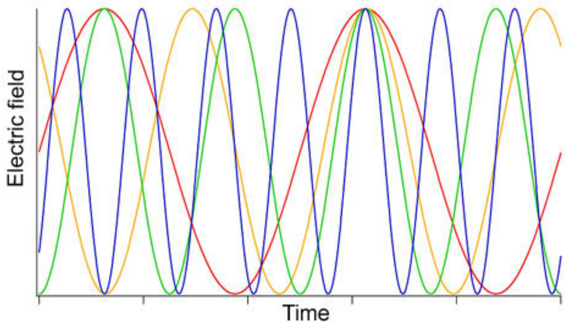
$$B_{max} = \frac{193.4 \cdot 10^{12}}{100} = 1.934 \cdot 10^{12} = 1.934 \text{ Tbit/s}$$

The number of audio channels is therefore:

$N_{max} = \frac{1.934 \cdot 10^{12}}{64 \cdot 10^{3}} \approx \mathbf{3 \cdot 10^7}$ **channels**

Exercise 2

One technique to make short pulses of light directly out of a cavity is called 'modelocking'. The different longitudinal modes of the cavity have a fixed phase relationship between each other so that the electric fields add constructively in one position of the cavity and destructively everywhere else (see the cartoon illustration below, with an exaggerated scale – the different modes have different oscillation frequencies shown by the colors).



A titanium-sapphire laser has a gain bandwidth of approximately 213 nm centered at 800 nm.

(a) When $\Delta\nu \ll \nu_0$ you can use the linear relationship between $\Delta\nu$ and $\Delta\lambda$ using the differentiation approach. Write this equation.

We have that $\nu_0 = c/\lambda_0$. Therefore assuming that $\Delta\nu \ll \nu_0$ we get that :

$$\frac{|\Delta\nu|}{|\Delta\lambda|} \approx \frac{c}{\lambda_0^2}$$

$$\boldsymbol{|\Delta\nu| \approx \frac{c}{\lambda_0^2}|\Delta\lambda|}$$

(b) What is the minimum possible pulse duration assuming transform-limited pulses and a Gaussian pulse shape. Use the linear approximation.

We have for a transform-limited Gaussian pulse:

$$\Delta\nu\Delta\tau = 0.44$$

$$\Delta\tau = \frac{0.44\lambda_0^2}{c\Delta\lambda}$$

$$\boldsymbol{\Delta\tau = 4.4\ \mathrm{fs}}$$

(c) If the pulse is instead sech²-shaped, how does the answer change? Use the linear approximation.

IF the shape is sech²:

$$\Delta\nu\Delta\tau = 0.315$$

$$\boldsymbol{\Delta\tau = 3.15\ \mathrm{fs}}$$

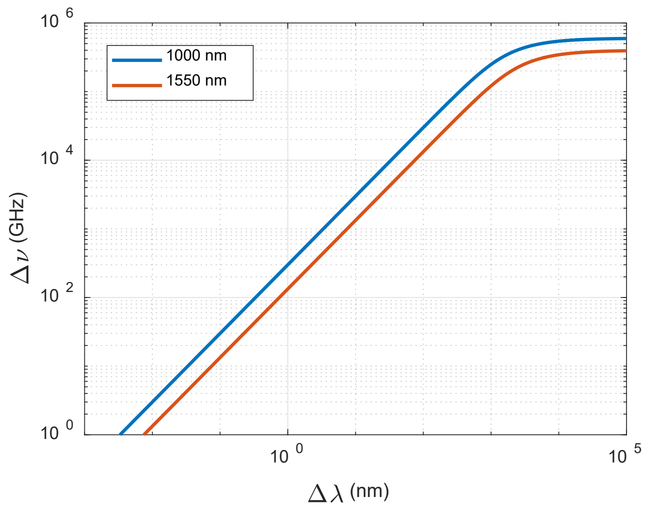
(d) Derive the exact expression for $\Delta\nu$ as a function of $\Delta\lambda$, and the central wavelength λ_0

We have that :

$$\Delta\lambda = \frac{c}{\nu_0 - \frac{\Delta\nu}{2}} - \frac{c}{\nu_0 + \frac{\Delta\nu}{2}}$$

$$\boldsymbol{\Delta\lambda = \frac{\lambda_0^2}{c} \frac{\Delta\nu}{1 - \left(\frac{\Delta\nu}{2\nu_0}\right)^2}}$$

(e) Plot (using matlab), $\Delta\nu$ vs $\Delta\lambda$ for $\Delta\nu$ between 1 GHz and 10^6 GHz, and for $\lambda_0 = 0.5\ \mu\text{m}$ $1\ \mu\text{m}$ and $1.5\ \mu\text{m}$ (use a log-log to see better)



(f) From the graph, show that a Gaussian 2 fs pulse at $1\ \mu\text{m}$ requires a bandwidth approximately 850 nm.

The bandwidth for such a pulse is:

$$\Delta\nu = \frac{0.44}{\Delta\tau} = 220\ \text{THz}$$

Using the graph (or you can use the full expression given that now $\Delta\nu \ll \nu_0$ does not hold), we read that the bandwidth is indeed close to 850 nm (goes from the visible to the NIR !)

Exercise 3

A 1550 nm digital communication system operates at a rate of 1 Gb/s. It receives an average of - 40 dBm at the output of the link. It is assumed that '1' and '0' bits are equally likely to occur (that is there is 50% of the time a bit '1' is received and the other 50% a bit '0' is received). You can also assume that a bit '0' contains no power.

(a) What is the power of a '1' bit in dBm and in mW?

Since '0' and '1' bits are equally probable, with P_0 and P_1 of power respectively, the average power P_{avg} is given by:

$$P_{avg} = \frac{(P_1 + P_0)}{2}$$

We assume that '0' bits contain no power, i.e. $P_0 = 0$, we therefore get that the power of a '1' is twice the average power. In dB scale, doubling corresponds to an approximate increase of 3 dB. Therefore:

$$P_1 = -40 + 3 = \mathbf{-37\ dBm}$$

In linear scale we get:

$$P_1 = 10^{(-37/10)} = \mathbf{0.1995\ \mu W}$$

(b) How many photons are received within each '1' bit ?

To obtain the number of incident photon at the receiver during a single '1' bit we divide the energy contained in a '1' bit by the energy of one photon. Let T_B be the duration of a bit. We can find T_B from the bit rate:

$$T_B = \frac{1}{B} = 1 \cdot 10^{-9}\ s$$

The energy in the bit is :

$$E_{bit} = P_1 T_B = 1.995 \cdot 10^{-16}\ \mathrm{J}$$

Given that the energy of a 1550 nm photon is $1.282 \cdot 10^{-19}$ J, we get the number of photons:

$$N = \frac{E_{bit}}{1.282 \cdot 10^{-19}} \approx \mathbf{1556\ photons}$$

Exercise 4

We need to estimate the coherence length for both LED. To do so we must get the following information from the spectra: central wavelength, 3-dB bandwidth (bandwidth at the FHMW) and the shape.

LED 1:

- Central wavelength: $\lambda_c = 0.83\ \mu\mathrm{m}$
- $\Delta\lambda = 18\ \mathrm{nm}$
- Shape : estimated to be Gaussian

The coherence length is given by :

$$l_c = c\tau_c = c\left(\frac{0.66}{\Delta\nu}\right)$$

$$l_c \approx c\left(0.66\frac{\lambda_c^2}{c\Delta\lambda}\right)$$

$$l_c \approx \left(0.66\frac{\lambda_c^2}{\Delta\lambda}\right)$$

$$\boldsymbol{l_c \approx 25.26\ \mu m}$$

LED 2:

- Central wavelength: $\lambda_c = 1.32\ \mu\text{m}$
- $\Delta\lambda = 47.6\ \text{nm}$
- Shape : estimated to be rectangular

The coherence length is given by :

$$l_c = c\tau_c = c\left(\frac{1}{\Delta\nu}\right)$$
$$l_c \approx c\left(\frac{\lambda_c^2}{c\Delta\lambda}\right)$$
$$l_c \approx \left(\frac{\lambda_c^2}{\Delta\lambda}\right)$$
$$\boldsymbol{l_c \approx 36.6\ \mu\text{m}}$$

Despite the wider bandwidth, LED 2 has a better coherence mainly due to its operating wavelength as well as the shape.

Exercise 5

A ruby laser makes use of a 10 cm long ruby rod (n = 1.76), has a transition cross section is $\sigma(\nu_0) = 1.26 \cdot 10^{-20}\ \text{cm}^2$, and operates on this transition at 694.3 nm. Both ends of the ruby rod are polished and coated so that each has a reflectance of 80%. Assume that there are no scattering or other extraneous losses.

As this laser is pumped, the medium goes from absorbing to amplifying. What is the required threshold population difference required to reach laser oscillation?

The required condition to start laser oscillation is when the gain coefficient of the medium exceeds the loss due to the feedback. Therefore:

$$\sigma(\nu_0)N_{th} > \frac{1}{2d}\ln\frac{1}{R^2}$$

Given the laser parameters:

$$N_{th} > \frac{1}{1.26 \cdot 10^{-20}} \cdot \frac{1}{20} \cdot \ln\frac{1}{0.64}$$
$$\boldsymbol{N_{th} > 1.77 \cdot 10^{18}\ \text{cm}^{-3}}$$

Exercise 6

In class, we derived that when lasing, the population difference inside a laser is clamped to: $N_{th} = \frac{1}{a\tau_p}$, with a a quantity associated with the probability that a carrier will capture a photon and give rise to stimulated emission. We want to find an expression for the quantity a.

(a) When a laser starts lasing, what happens to the gain?

When the laser starts lasing, the gain equals the loss of the cavity. Using the known expression for gain and loss, we can write:

$$\gamma(\nu) = \alpha_r$$

$$\sigma(\nu)N_{th} = \frac{1}{c\tau_p}$$

(b) Using the answer from part (a), show that the quantity a is given by $a = \sigma(\nu_0)c$: where $\sigma(\nu_0)$ is the transition cross section and c the speed of light in the laser medium.

We can now find an expression for the quantity a since:

$$a = \frac{1}{N_{th}\tau_p}$$

From the equation in part (a), we therefore have:

$$\boldsymbol{a = \sigma(\nu)c}$$

Exercise 7

A mode-locked laser produces pulses with an average power of 1 W, a repetition rate of 100 MHz, and a pulse duration of 50 fs.

(a) What is the energy per pulse?

By definition:

$$E_p = \frac{P_{avg}}{f_{rep}} = \frac{1}{100 \cdot 10^6} = 10 \text{ nJ}$$

(b) What is the peak power of the pulse?

By definition:

$$P_{peak} = \frac{E_p}{\Delta\tau} = \frac{10 \cdot 10^{-9}}{50 \cdot 10^{-15}} = 200 \text{ kW}$$